MATH 1300, Mathematical Explorations

Pattern, Conjecture, Proof

Activity

Divide the class into groups; give each group a "visual proof" (in resources). As them to discuss the proof and figure it out. Then have a student from each group stand up to explain the proof to the rest of the class. Discuss afterwards whether these proofs were satisfactory: what was, what wasn't, how it could be improved. Have students discuss whether this makes them believe/understand the claims they were "proving" better.

It's important to mention that the question "what is a proof?" is one that leads to argument within the mathematical community, as well. The link "PWW and Mathematical Proof" below leads to a discussion on the MAA website about whether proofs without words are convincing, whether they count as proof, and what their modes of failure can be. Both students who are convinced by the proofs without words and students who aren't (for whatever reason!) should feel secure that there are likely people within the mathematical community who share their views.

Questions for class

- What is a proof?
- What is the goal of a proof?
- What makes a proof convincing? unconvincing?
- What makes a proof "correct" or "incorrect"?
- Does a mistake in a proof imply that the result is incorrect?

References and resources

The Art of Mathematics: Truth, Reasoning, Certainty, and Proof

The Art of Mathematics: Patterns

Proofs without Words: Exercises in Visual Thinking, by Roger Nelsen.

MAA: Proofs without Words 2.0

MAA: PWW and Mathematical Proof

Notes

Assignments

1. Define a sequence where the nth term is the last digit of n^2. So the sequence starts $1, 4, 9, 6, 5, 6, 9, \ldots$ Write down the first 20 terms in the sequence. Is there a pattern to these? Prove that the pattern you see continues forever.
2. Download the patterns book, and do (and turn in) Investigations 32-36 on page 37. Do you think that this works to prove that there are two people with the same number of hairs on their heads? Do you trust this argument? (Expected length: 2 paragraphs)
3. Do Investigations 21-23 in Chapter 5 of the reasoning book.

Follow-on activities